Roll No. ....

DD–2802

M. A./M. Sc. (Previous)
EXAMINATION, 2020

MATHEMATICS

Paper Second

(Real Analysis)

Time : Three Hours

Maximum Marks : 100

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) If r' is continuous on $[a, b]$, then prove that r is rectifiable and :

$$\Lambda(r) = \int_a^b |r'(t)|\, dt$$

(b) State and prove fundamental theorem of calculus.

(c) Let f be monotonic on $[a, b]$ and let α be continuous and monotonically increasing on $[a, b]$. Then prove that :

$$f \in R(\alpha).$$

Unit—II

2. (a) Show that the sequence $\{f_n\}$, where $f_n(x) = nx(1-x)^n$ does not converge uniformly on [0, 1].

   (b) In a power series $\sum_{n=0}^{\infty} a_n x^n$ converges at the end point $x = \mathrm{R}$ of the interval of convergence (– R, R), then prove that it is uniformly convergent in the closed interval [0, R].

   (c) State and prove Weierstrass's M-test for uniform convergence as series.

Unit—III

3. (a) Prove that a linear operator A on a finite dimensional vector space X is one to one if and only if the range of A is all of X, that is if and only if A is onto.

   (b) Find the maximum and minimum values of the function :

$$f(x, y) = 2x^2 - 3y^2 - 2x,$$

   subject to the constraint $x^2 + y^2 \le 1$.

   (c) Let Ω be the set of all invertible linear operators on R^n. if $\mathrm{A} \in \Omega$, $\mathrm{B} \in \mathrm{L}(\mathrm{R}^n)$ and $\|\mathrm{B} - \mathrm{A}\| \|\mathrm{A}^{-1}\| < 1$, then prove that $\mathrm{B} \in \Omega$.

Unit—IV

4. (a) State and prove Lebesgue's monotone convergence theorem.

   (b) Show that "A Borel measurable set is Lebesgue measurable."

   (c) Let $\{\mathrm{A}_n\}$ be a countable collection of sets of real numbers. Then show that :

$$m^*\left(\bigcup_{n=1}^{\infty} \mathrm{A}_n\right) \le \sum_{n=1}^{\infty} m^*(\mathrm{A}_n).$$

Unit—V

5. (a) State and prove Minkowski's inequalities for L^p -spaces.

   (b) State and prove Jensen's inequality.

   (c) Show that the class **B** of all μ^*-measurable sets is a σ-algebra of subsets of X. If $\bar{\mu}$ is μ^* restricted to **B**, then $\bar{\mu}$ is a complete measure on **B**.