Roll No. ....

DD–771

M. A./M. Sc. (Fourth Semester) EXAMINATION, 2020

MATHEMATICS

(Optional—B)

Paper Fifth

(Graph Theory)

Time : Three Hours

Maximum Marks : 80

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) For any $s, t \geq 1$, prove that there is $R(s, t) < \infty$ such that any graph on $R(s, t)$ vertices contains either an independent set of size s or a clique of size t. In particular :

$$R(s,t) \leq \binom{s+t-2}{s-1}.$$

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(b) Prove that every graph on $\begin{bmatrix} k+l \\ k \end{bmatrix}$ vertices contains either a complete sub-group on $k+1$ vertices or an independent set of $l+1$ vertices.

(c) Prove that for any $s \geq 2$,

$$R(s, s) \geq 2^{\frac{s}{2}}.$$

Unit—II

2. (a) Prove that every vertex of a composite connected graph lies on a 4-cycle.

(b) Prove that the line-group and the point group of a graph G are isomorphic iff G has at most one isolated point and k_2 is not a component of G.

(c) Prove that the group of the union of two graphs is the sum of their groups :

$$\Gamma(G \cup G_2) \equiv \Gamma(G_1) + \Gamma(G_2)$$

iff no component of G, is isomorphic with a component of G_2.

Unit—III

3. (a) Prove that every 3-chromatic maximal planar graph is uniquely 3-colorable.

(b) If $B_1, B_2, ..., B_r$ are the blocks of a graph G, then prove that :

$$\phi(G, x) = \frac{1}{x^{r-1}} \prod_{i=1}^{\infty} Q(B_i, x).$$

(c) Prove that the coefficients of every chromatic polynomial alternate in sign.

Unit—IV

4. (a) Prove that each cycle $C_n, n \geq 3$ is chromatically unique.

   (b) Prove that for any graph G :

   $$\chi(G) \leq 1 + \max \delta(G'),$$

   where the maximum is taken over all induced subgroups G' of G.

   (c) Prove that for a connected graph G,

   $$T(G, 1, 1) = \tau(G),$$

   the number of spanning trees of G.

Unit—V

5. (a) Prove that a graph G is *n*-line connected iff every pair of points are joined by at least *n*-line disjoint paths.

   (b) State and prove Menger's theorem for digraph (vertex form).

   (c) Prove that every acyclic graph without isolated has a unique basis consisting of its transmitters.