Roll No. ....

DD–2806

M. A./M. Sc. (Final) EXAMINATION, 2020

MATHEMATICS

(Compulsory)

Paper First

(Integration Theory and Functional Analysis)

Time : Three Hours

Maximum Marks : 100

Note : Attempt any *two* parts from each question. All questions carry equal marks.

1. (a) State and prove Radon-Nikodym theorem.

   (b) Let E be a measurable set such that $0 < \nu(E) < \infty$, then prove that there is a positive set A contained in E with $\nu(A) > 0$.

   (c) State and prove Caratheodory extension theorem.

2. (a) State and prove Riesz-Markoff theorem.

   (b) Define Baire set and prove that every compact Baire set is a G_δ set.

   (c) If μ is finite Baire measures on the real line, then show that its cumulative distribution function F is monotone increasing bounded function which is continuous on the right, moreover $\lim_{n \to \infty} F(X) = 0$.

3. (a) Let $\{x_n\}$ be a weakly convergent sequence in a normed linear space X. Then prove that :

   (i) The weak limit of $\{x_n\}$ is unique.

   (ii) The sequence $\{\|x_n\|\}$ is bounded.

   (b) Let $c\ [a,\ b]$, be a function space of real valued continuous functions defined on $[a,\ b]$. Define $\|\cdot\|_\infty$ by $\|\cdot\|_\infty : c[a,b] \to R$, where $\|f\|_\infty = \max_{t\in[a,b]} |f(t)|$, then prove that $(c[a,b], \|\cdot\|_\infty)$ is Banach space.

   (c) State and prove Riesz lemma.

4. (a) Prove that a Banach space is reflexive if and only if its dual space is reflexive.

   (b) State and prove closed graph theorem.

   (c) State and prove Uniform Boundedness theorem.

5. (a) State and prove the generalized Lax-Milgram theorem.

   (b) Prove that product of two bounded self-adjoint operators S and T on a Hilbert space H is self-adjoint iff the operators commute i. e. if ST = TS.

   (c) A Banach space is Hilbert space iff its norm satisfies the parallelogram law.