Roll No. ....

DD–2805

M. A./M. Sc. (Previous)

EXAMINATION, 2020

MATHEMATICS

Paper Fifth

(Advance Discrete Mathematics)

Time : Three Hours

Maximum Marks : 100

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Define Tautology. If H_1, H_2, ..., H_m and P imply Q, then prove that $H_1, H_2, ..., H_m$ imply $P \to Q$.

   (b) Define Semigroup Homomorphism. Let (S, *), (T, Δ) and (V, $\oplus$) be semigroups and $g : S \to T$ and $h : T \to V$ be semigroup homomorphism. Then show that $(h \circ g) : S \to V$ is a semigroup homomorphism from (S, *) to (V, $\oplus$).

   (c) Show that :

$$P \to (Q \to R) \Leftrightarrow P \to (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \to R$$

Unit—II

2. (a) Define distributive lattice and let (L, *, $\oplus$) be a distributive lattice, then prove that for any $a, b, c \in L$:

$$(a * b = a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c.$$

   (b) Use the Karnaugh map representation to find a minimal sum-of-product expression of the following function :

$$f(a, b, c, d) = \sum (10, 12, 13, 14, 15).$$

   (c) Define a lattice and sublattice. Prove that the set

$$M = \{1, 2, 3, 4, 6, 8, 12, 24\};$$

   the set of all divisors of the integer 24 is a sublattice of the lattice (1, $\leq$) with respect to the relation "$\leq$" where :

$$L = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$$

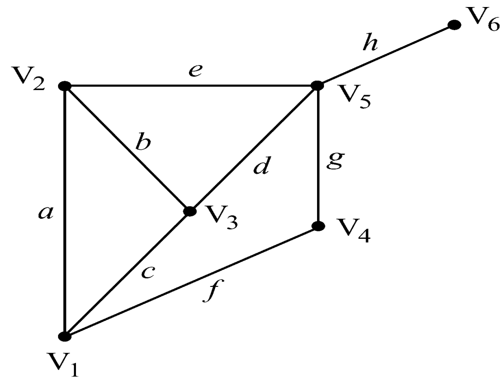
   and "$x \leq y$" means x divides y.

Unit—III

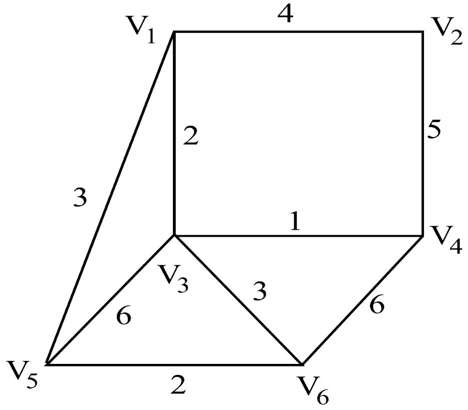
3. (a) Define planar graph and for any connected planar graph, prove that :

$$V - e + r = 2.$$

   (b) Define Incidence matrix and find the incidence matrix in given graph :



(c) Define spanning tree and find the minimal spanning tree for the weighted graph in the following figure using Kruskal's algorithm :



Unit—IV

4. (a) Define transition system. Prove that for any transition function δ and for any two input strings x and y :

$$\delta(q, xy) = \delta(\delta(q, x), y).$$

(b) Define Mealy machine and consider the Moore machine described by the transition table given by table. Construct the corresponding Mealy machine :

Moore Machine

Present State	Next State		Output
	$a = 0$	$a = 1$	
$\rightarrow q_1$	q_1	q_2	0
q_2	q_1	q_3	0
q_3	q_1	q_3	1

(c) Define the following :

(i) Equivalence of finite state machine

(ii) Reduced machine

(iii) Deterministic finite automata

(iv) Non-deterministic finite automata

Unit—V

5. (a) Define Polish Notation and prove that the rank of any well formed Polish formula is 1 and the rank of any proper head of a polish is greater than or equal to 1.

(b) State and prove Pumping Lemma.

(c) Define Language and show that the language $L(G) = \{a^n b a^n : n \geq 1\}$ is generated by grammar :

$$G = \{(S, c), (a, b), S, \phi\},$$

where ϕ is the set of production $S \rightarrow aca, c \rightarrow aca, c \rightarrow b$.