Roll No. ....

DD–2814

M. A./M. Sc. (Final) EXAMINATION, 2020

MATHEMATICS

(Optional)

Paper Fifth (*iii*)

(Fuzzy Sets and Their Applications)

Time : Three Hours

Maximum Marks : 100

Note : Attempt any *two* parts from each question. All questions carry equal marks.

1. (a) Prove that a Fuzzy set A on R is convex iff :

$$A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)]$$

for all $x_1, x_2 \in R$ and all $\lambda \in [0,1]$ where min denotes the minimum operator.

(b) Let $f : X \to Y$ be an arbitrary crisp function. Then for any $A \in \mathbf{F}(\alpha)$ and all $\alpha \in [0,1]$ prove that (f fuzzified by the extension principle) $\alpha[f(A)] \supseteq f({}^{\alpha}A)$ but not conversely.

(c) Prove that the standard fuzzy intersection is the only idempotent *t*-norm. Also prove that (ab, $a + b - ab$, (c_s) is a dual triple where (c_s) is the standard complement.

2. (a) Calculate the fuzzy cardinality and scalar cardinality of the fuzzy set defined by A : X → [0, 1], where $X = \{0, 2, 4, 6, ..., 80\}$ and :

$$A(x) = \begin{cases} 0 & \text{if } x \leq 20 \text{ or } x \geq 60 \\ (x-20)/15 & \text{if } 20 < x < 35 \\ (60-x)/15 & \text{if } 45 < x < 60 \\ 1 & \text{if } 35 \leq x \leq 45 \end{cases}$$

(b) Let $A \in \mathbf{F}(R)$. Then prove that A is a fuzzy number iff there exists a closed interval $[a,b] \neq \phi$ such that :

$$A(x) = \begin{cases} 1 & \text{for } x \in [a,b] \\ l(x) & \text{for } x \in (-\infty, a) \\ r(x) & \text{for } x \in (b, \infty) \end{cases}$$

where l is a function from $(-\infty, a)$ to $[0,1]$ that is monotonic increasing, continuous from the right and such that $l(x) = 0$ for $x \in (-\infty, w_1)$ and r is a function from (b, ∞) to $[0,1]$ that is monotonic decreasing, continuous from the left and such that $r(x) = 0$ for $x \in (w_2, \infty)$.

(c) Find the transitive max-min closure R_T for a fuzzy relation defined by the following membership matrix :

$$R = \begin{bmatrix} .8 & .6 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ .2 & .4 & 0 & 0 \\ 0 & 0 & .9 & .8 \end{bmatrix}$$

Unit—III

3. (a) Solve p o Q = r, where :

$$Q = \begin{bmatrix} .1 & .4 & .5 & .1 \\ .9 & .7 & .2 & 0 \\ .8 & 1 & .5 & 0 \\ .1 & .3 & .6 & 0 \end{bmatrix} \text{ and } r = [\ .8\ .7\ .5\ 0\ ]$$

(b) Prove that :

$$Pl(A) \geq Bel\ (A)$$

for all $A \in P(X)$.

(c) Let a fuzzy set F be defined on N by F = .4/1 + .7/2 + 1/3 + .8/4 + .5/5 and $A(x) = 0$ for all $x \notin \{1, 2, 3, 4, 5\}$. Determine Nec (A) and Pos (A) induced by F for all A ∈ P ({1, 2, 3, 4, 5}).

Unit—IV

4. (a) Write short notes on the following :

(i) Multivalued logic

(ii) Linguistic hedges

(b) Let sets of values of variables x and y be $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$, respectively. Assume that a proposition "if x is A, then y is B" is given, where $A = .5/x_1 + 1/x_2 + .6/x_3$ and $B = 1/y_1 + .4/y_2$. Then, given a fact expressed by the proposition "x is A′," where $A' = .6/x_1 + .9/x_2 + .7/x_3$. Use the generalized modus ponens to derive a conclusion in the form "y is B`."

(c) Write the reasonable axioms of fuzzy implications.

Unit—V

5. (a) What are the basic components of a general fuzzy controller ? Discuss in detail.

(b) Assume that each individual of a group of eight decision makers has a total preference ordering P; ($i \in N_8$) on a set of alternatives $X = \{w, x, y, z\}$ as follows :

$$P_1 = \{w, x, y, z\}$$
$$P_2 = P_5 = \{z, y, x, w\}$$
$$P_3 = P_7 = \{x, w, y, z\}$$
$$P_4 = P_8 = \{w, z, x, y\}$$
$$P_6 = \{z, w, x, y\}$$

Then :

(i) Find the fuzzy preference ordering relations.

(ii) Also find α-cuts of this fuzzy relation S and group level of agreement concerning the social choice denoted by the total ordering $\{w, z, x, y\}$.

(c) Solve the following fuzzy linear programming problem :

Max. :

$$z = 6x_1 + 5x_2$$

s. t. :

$$(5, 3, 2)x_1 + (6, 4, 2)x_2 \leq (25, 6, 9)$$
$$(5, 2, 3)x_1 + (2, 1.5, 1)x_2 \leq (13, 7, 4)$$
$$x_1, x_2 > 0.$$