Roll No. .....

DD–2812

M. A./M. Sc. (Final) EXAMINATION, 2020

MATHEMATICS

Paper Fifth (*i*)

(General Relativity and Cosmology)

Time : Three Hours

Maximum Marks : 80

Note : Attempt any *two* parts from each Unit. All questions carry equal marks.

Unit—I

1. (a) Define covariant derivative of a contravariant vector and show that it is a tensor.

   (b) If :

$$A_{ij} = A_{i,j} - A_{j,i}$$

   prove that :

$$A_{ij,k} + A_{jk,i} + A_{ki,j} = 0$$

   (c) If A_{ik} is an antisymmetric tensor of the second order, show that :

$$\frac{\partial A_{ik}}{\partial x^m} + \frac{\partial A_{km}}{\partial x^i} + \frac{\partial A_{mi}}{\partial x^k}$$

   is a tensor.

Unit—II

2. (a) Prove Bianchi identity :

$$R^a_{ijk,l} + R^a_{ikl,j} + R^a_{ilj,k} = 0$$

   (b) State Einstein principle of equivalence. What are the observable consequences of general theory of relativity ?

   (c) Obtain the general relativistic equation of motion of a particle in gravitational field through a principle of least action.

Unit—III

3. (a) Derive Schwarzchild exterior solution for the gravitational field of a single mass continually at rest at the origin.

   (b) Derive the formula for energy momentum tensor for a perfect fluid in the form :

$$T^\nu_\mu = (\rho + p)\, v_\mu v^\nu - g^\nu_\mu p$$

   (c) Obtain the equation for a planetary orbit and obtain expression for the advance of the perihelion to an orbit.

Unit—IV

4. (a) Obtain the line element for Einstein and de-Sitter's cosmological models.

   (b) Obtain the line element for Robertson's non-statics cosmological model. Show how this model reveals the universe is expanding.

   (c) Discuss red shift in Robertson-Walker line element.

Unit—V

5. (a) Obtain expression for particle horizon for Einstein-de-Sitter universe.

   (b) State and explain Einstein-de-Sitter universe model.

   (c) Discuss Eddington's-Lamaitre cosmological model with cosmological constant $\wedge$.