Roll No. ....

DD–2810

M. A./M. Sc. (Final) EXAMINATION, 2020

MATHEMATICS

(Optional)

Paper Fourth (*i*)

(Operations Research)

Time : Three Hours

Maximum Marks : 100

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Use Simplex method to :

Minimize :

$$z = x_2 - 3x_3 + 2x_5$$

Subject to the constraints :

$$3x_2 - x_3 + 2x_5 \le 7$$

$$-2x_2 + 4x_3 \le 12$$

$$-4x_2 + 3x_3 + 8x_5 \le 10$$

$$x_2 \ge 0$$

$$x_3 \ge 0 \text{ and } x_5 \ge 0 .$$

(b) Use duality to solve the following L. P. P. :

Maximize :

$$z = 2x_1 + x_2$$

Subject to the constraints :

$$x_1 + 2x_2 \le 10$$

$$x_1 + x_2 \le 6$$

$$x_1 - x_2 \le 2$$

$$x_1 - 2x_2 \le 1$$

$$x_1, x_2 \ge 0.$$

(c) For the following L. P. P. :

Maximize :

$$z = (3 - 6\lambda)x_1 + (2 - 2\lambda)x_2 + (5 + 5\lambda)x_3$$

Subject to the constraints :

$$x_1 + 2x_2 + x_3 \le 430$$

$$3x_1 + 2x_3 \le 460$$

$$x_1 + 4x_2 \le 420$$

$$x_1, x_2, x_3 \ge 0.$$

Find the range of λ over which the solution remains basic feasible and optimal.

Unit—II

2. (a) Use Vogel's approximation method to obtain an initial basic feasible solution of the Transportation problem :

$$\begin{array}{cc} & \begin{array}{cccc} D & E & F & G \end{array} & \text{Available} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 11 & 13 & 17 & 14 \\ 16 & 18 & 14 & 10 \\ 21 & 24 & 13 & 10 \end{bmatrix} & \begin{array}{c} 250 \\ 300 \\ 400 \end{array} \\ \text{Demand} & \begin{array}{cccc} 200 & 225 & 275 & 250 \end{array} & \end{array}$$

(b) A departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given in the matrix below :

Tasks	Men			
	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize the total man-hours ?

(c) A project consists of a series of tasks labelled A, B, ..., H, I with the following relationships (W < X, Y means X and Y cannot start until W is completed;

X, Y < W means W cannot start until both X and Y are completed). With this notation construct the network diagram having the following constraints :

A < D, E_j B, D < F_j C < G; B, G < H; F, G < I.

Find also the minimum time of completion of the project, when the time (in days) of completion of each task is as follows :

Task	Time
A	23
B	8
C	20
D	16
E	24
F	18
G	19
H	4
I	10

Unit—III

3. (a) Use dynamic programming to show that :

$$z = P_1 \log P_1 + P_2 \log P_2 + ... + P_n \log P_n$$

s. t. c. :

$$P_1 + P_2 + P_3 + ... + P_n = 1 \text{ and } P_j \geq 0$$

$$(j = 1, 2, ..., n)$$

is a minimum when $P_1 = P_2 = ... = P_n = \frac{1}{n}$.

(b) Obtain the optimal strategies for both persons and the value of the game for zero-sum two-person game whose payoff matrix is as follows :

$$\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

(c) Use Branch and Bound method to solve the following L. P. P. :

Maximize :

$$z = 7x_1 + 9x_2$$

s. t. c. :

$$-x_1 + 3x_2 \leq 6$$

$$7x_1 + x_2 \leq 35$$

$$x_2 \leq 7$$

$$x_1, x_2 \geq 0$$

and are integers.

Unit—IV

4. (a) Solve the non-linear programming problem :

Optimize :

$$z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1 x_2$$

s. t. c. :

$$x_1 + x_2 + x_3 = 15$$

$$2x_1 - x_2 + 2x_3 = 20.$$

(b) Use Wolfe's method to solve the QPP :

Maximize :

$$z = 2x_1 + 3x_2 - 2x_1^2$$

s. t. c. :

$$x_1 + 4x_2 \le 4$$

$$x_1 + x_2 \le 2$$

$$x_1, x_2 \ge 0.$$

(c) Use Separable convex programming to solve the NLPP :

Maximize :

$$f(x) = 3x_1 + 2x_2$$

s. t. c. :

$$g(x) = 4x_1^2 + x_2^2 \le 16 \text{ and } x_1, x_2 \ge 0.$$